Exploration of Rose Curves with NetPad

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Abstract. Dynamic geometry system has been promoted actively in mathematical education because of its potential to comprehensively improve the core mathematical literacy required in an increasingly technology-based society. The ability to dynamically drag geometric elements to compare and discover relationships, and the ability to control movements and observe trajectory, are what make dynamic geometry system a powerful and efficient mathematical learning tool. This paper focuses on the application of dynamic geometry systems in inquiry-based maths education in middle school. Specifically, an inquiry-based learning activity named "mathematically drawing flowers with NetPad" is designed for students to study a characteristic feature of rose curves, i.e., the relationship between the number of petals and coefficient of rose curve equation. By sequentially graphing equations $\rho = A \cdot cos(n\theta)$ and $\rho = A \cdot cos(n\theta) + B$ with NetPad in a constructive way, and then observing, reasoning, verifying, and expressing, students can closely experience a complete generating process of maths knowledge, and further improve their innovation awareness and practical ability.

Keywords: Rose Curve · Dynamic Geometry · NetPad · Inquiry-Based Learning

1 Introduction

As a closed curve with various shapes, the rose curve has fascinated people since it was first studied by the Italian mathematician Guido Grandi in the 1700s. Moreover, mathematical properties and beauties of the rose curve have been practically applied in current industrial operations, such as orbital forming pressing, rope braiding, scanning, and weaving [1, 2]. Hence, topics about the rose curve gradually draw the attention of mathematical education in middle schools, especially inquiry-based mathematical education.

Instead of the lecture-based instructional approach, an inquiry-based mathematical learning activity refers to a student-centered and teacher-directed paradigm of teaching mathematics, in which students are expected to gain mathematical knowledge and further cultivate their innovation awareness by closely experiencing the generating process

W. Hong and Y. Weng (Eds.): ICCSE 2022, CCIS 1812, pp. 122–135, 2023.
https://doi.org/10.1007/978-981-99-2446-2_12

of mathematical knowledge, that is, finding and proposing meaningful mathematical problems, making a reasonable maths hypothesis, giving plans to solve problems, and arguing conclusions with mathematical ways [3–5]. According to a literature search, inquiry-based rose curve learning activities are mainly carried out by using physical tools or information technology. For example, Gao [6] proposes a STEM case in that students are guided to explore and physically draw a 4-petalled rose curve; Xu [7] proposes a case to explore the relationship between the number of petals and coefficient n of equation $\rho = A \cdot sin(n\theta)$ with a maths software; and Tuyetdong [8] further expands the exploration by drawing various rose curves in a dynamic geometry system. The maths nature of the 4-petalled rose curve is well demonstrated in Gao's case, and students are suggested to further draw other rose curves according to the same exploring method, it is an excellent example for applying the rose curve but not an efficient way to explore various rose curves because the drawing cost is relatively high. While in Xu and Tuyetdong's cases, properties of the rose curve are explored by observing different graphs, and graphs can be quickly plotted by introducing an equation and manipulating variables in dynamic geometry systems; although information technologies help saving more efforts, skipping the process and directly showing results after changing variables make the maths nature of the curves being hidden, and benefit of information technology tools are not fully utilized to affect learning behaviors [9–11].

In fact, the ability of dynamically dragging an object to compare and discover relationships while keeping the geometric constraints of the object unchanged is what makes a dynamic geometry system a powerful mathematical learning tool. In addition, users are allowed to interact with geometric objects and receive immediate visual responses to their actions; the interaction is beneficial in understanding, and the instant feedback of visualizing students' ideas and confirming or falsifying their assumptions will make problem solving more efficient [12, 13]. Hence, the availability of a dynamic geometry system allows some constructive ways that use the properties of rose curves to draw them. In particular, as a well-designed dynamic geometry system, NetPad has been widely accepted by middle school students and their teachers, barriers of technology acceptance are relative lower; moreover, the browser-based dynamic geometry system can be accessed from various terminals, which makes NetPad a convenient leaning tool [14–16].

Hence, this paper focuses on the application of dynamic geometry systems in inquiry-based maths education in middle school. A specific inquiry-based learning activity named "mathematically drawing flowers with NetPad" is designed for students to study a characteristic feature of rose curves, i.e., the number of petals. By graphing equations with NetPad in a constructive way, and then observing, reasoning, verifying and expressing, students are expected to closely experience the generating process of mathematical knowledge, and further cultivate their innovation awareness and practical ability. On the basis of students' prior knowledge about the polar coordinate system and cosine curve in the rectangular coordinate system, this activity is designed in the following four sections.

2 First Impression of Rose Curves

At the very beginning, teachers may assign a bridge-in task, i.e., observing and collecting shapes of flowers that students encounter in real life, expecting that they can extract several basic shapes similar to rose curves. Then, the rose curve is introduced as a tool for students to mathematically draw flowers having a desired number of petals.

Instead of directly providing static graphs or equations of rose curves, a more eye-catching way to impress and motivate students for subsequent learning is to present a generating process of a 4-petalled rose curve by means of a ladder model constructed in NetPad. As the course ware[1] shown in Fig. 1, radii *OA* and *OB* of circle *O* are perpendicular to each other; segments *CD*, *CE*, and *OF* are respectively perpendicular to segments *OA*, *OB*, and *ED*, then the trajectory of point F formed by dragging point *C* along circle *O* is the 4-petalled rose curve.

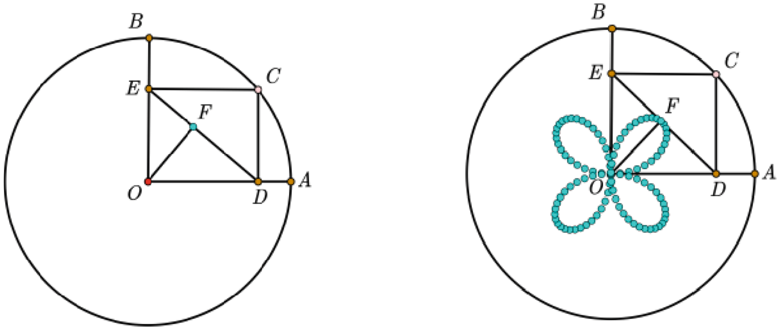

Fig. 1. Dynamically generating 4-petalled rose curve by dragging point *C*

On the basis of the model, there is a chance for students to creatively try and find how to change the number of petals by adjusting the model, such as what if radii *OA* and *OB* of circle *O* are not perpendicular to each other. Then, students can be encouraged to find the equation of the curve in polar coordinates. With point *O* as the pole, *OA* as the pole axis, $\angle AOF = \theta$, and it would not be difficult to get $OF = OD \cdot cos(\theta) = ED \cdot sin(\theta) \cdot cos(\theta) = OA/2 \cdot sin(2\theta)$. Obviously, the length of segment *OA* and coefficient of θ are keys that affect the graph. Hence, students would be motivated to see whether we can draw flowers having various numbers of petals according to equation $\rho = A \cdot sin(k\theta)$.

3 Constructively Graphing with NetPad

3.1 Inspire Students to Guess Based on Existing Knowledge

Since the fact that sine curve in rectangular coordinates corresponds to the horizontal coordinate and the central angle of a point moving around a circle, a model can be constructed to intuitively and dynamically show the generating process of the sine curve. As the courseware[2] shown in Fig. 2, point *A* moves along unit circle *C*, point C' moves along the x-axis at the speed of the value of the central angle of point *A*, line l_1 passes through point C' and is perpendicular to the x-axis, l_2 passes through point *A* and is

[1] https://www.NetPad.net.cn/resource_web/course/#/479130.
[2] https://www.NetPad.net.cn/resource_web/course/#/494350.

parallel to the x-axis, and then, the trajectory of cross-point B driven by points A and O' is the sine curve, which can appear bit by bit with the animation function.

By recalling the aforementioned existing knowledge, students may naturally come up with a logical idea: Is the graph of polar equation can be constructed by some points moved in certain forms?

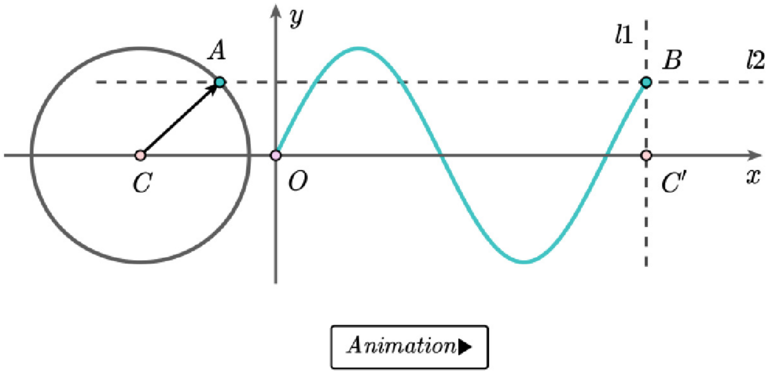

Fig. 2. Constructively graphing sine curve in rectangular coordinates

3.2 Analyze Equation

Students may directly analyze equation $\rho = A \cdot sin(k\theta)$, or analyze it after making some conversions. Because $\rho = A \cdot sin(k\theta) = A \cdot cos(k\theta - \pi/2)$, we know both $\rho = A \cdot sin(k\theta)$ and $\rho = A \cdot cos(k\theta)$ can be used to specify the rose curve, and a more generalized form is $\rho = A \cdot sin(\omega\theta + \varphi)$ or $\rho = A \cdot cos(\omega\theta + \varphi)$. For simpler calculation in the following part, here we suggest students adopt the polar equation $\rho = A \cdot cos(\omega\theta + \varphi)$.

Since students are more familiar with the rectangular coordinate system, they may wonder what happens if polar equation $\rho = A \cdot cos(\omega\theta + \varphi)$ is transformed into parametric equations in the rectangular coordinate system, that is:

$$\begin{cases} x = A \cdot cos(\omega\theta + \varphi) \cdot cos\theta \\ y = A \cdot cos(\omega\theta + \varphi) \cdot sin\theta \end{cases} \tag{1}$$

After being subjected to simple trigonometric conversions, Eq. (1) is organized into:

$$\begin{cases} x = \frac{A}{2}[cos((\omega + 1)\theta + \varphi) + cos((1 - \omega)\theta - \varphi)] \\ y = \frac{A}{2}[sin((\omega + 1)\theta + \varphi) + sin((1 - \omega)\theta - \varphi)] \end{cases} \tag{2}$$

The interesting thing is that parametric equations of two circular motions appear, as shown in Eq. (3) and Eq. (4). The two circular motions have the same radius, different moving speeds, and different phrase angles. That is, the graph of equation $\rho = A \cdot cos(\omega\theta + \varphi)$ is formed by a point in two different circular motions.

$$\begin{cases} x = \frac{A}{2}cos((\omega + 1)\theta + \varphi) \\ y = \frac{A}{2}sin((\omega + 1)\theta + \varphi) \end{cases} \tag{3}$$

$$\begin{cases} x = \frac{A}{2}cos((1 - \omega)\theta - \varphi) \\ y = \frac{A}{2}sin((1 - \omega)\theta - \varphi) \end{cases} \tag{4}$$

3.3 Construct Models

According to the geometric meaning of Eqs. (3) and (4), students can creatively construct a model to demonstrate the movement of a point driven by two circular motions in NetPad. A hint is the relative positions of two circular motions in the plane.

When points are running on two circles lying outside each other, the middle point of a segment connecting the two points satisfies our expectation; but the size of the graph seems not right due to a scaling effect. As the courseware[3] shown in Fig. 3, points A_1 and A_2 are respectively set to move according to Eqs. (3) and (4), then the trajectory of middle point M of segment A_1A_2 is a 4-petalled rose curve, but in the half size of the graph specified by the corresponding equations.

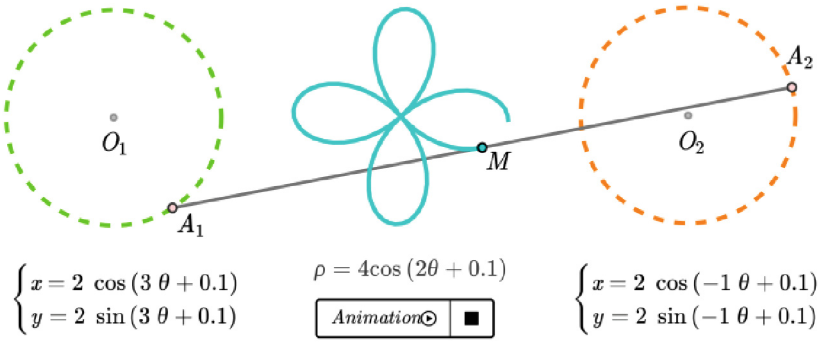

Fig. 3. A model based on separated circular motions

Then, students may consider a model containing tangent circular motions. As shown in Fig. 4, when circle O_2 externally or internally moves along circle O_1 in a tangent way, fixed point A on circle O_1 will make a circular motion too; and if fixed point A also moves along circle O_2, then point A is driven by two circular motions, and the movement of point A is just what we need. Obviously, no matter the externally tangent

[3] https://www.netpad.net.cn/resource_web/course/#/533753.

circular motions or internally tangent circular motions, the movement of the center of the running circle matters, leading that the tangent circular motions can be classified into secant circular motions, as shown in Fig. 4.

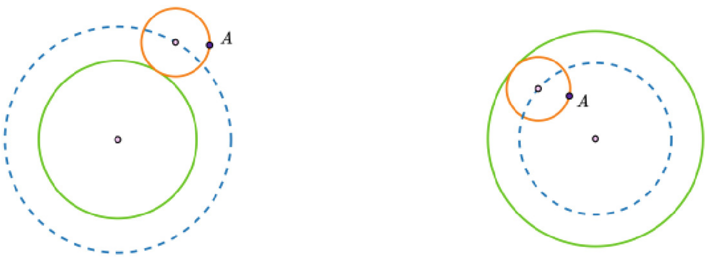

Fig. 4. A model based on tangent circular motions

More specifically, the external and internal movements can be rolling with or without slipping. For the case without slipping, it may not always draw the rose curve. In fact, this model is the key to the ladder model, students can try to find out the two circular motions hidden in the ladder model after this inquiry-based learning activity, and then, with the rule of the number of petals, various rose curve can be drawn according to the ladder model.

For the model of two secant circular motions, as the courseware[4] shown in Fig. 5, point O_2 moves along circle O_1 according to Eq. (3), point A moves along circle O_2 according to Eq. (4), and the trajectory of point A is the curve specified by the equation with specific parameters.

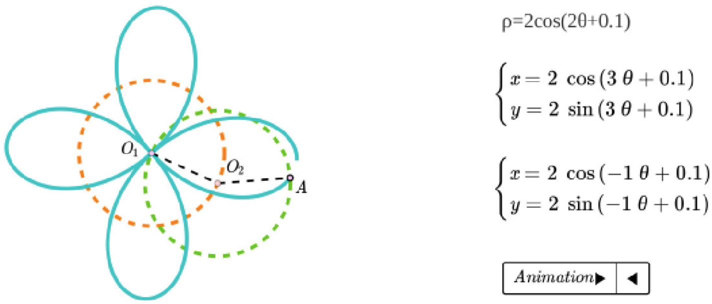

Fig. 5. A model based on secant circular motions

3.4 Parameter Settings

On the basis of the preliminary geometric construction, we will further set corresponding animation parameters by observing, analyzing, and expressing the ranges and geometric meanings of parameters A, ω, and φ.

4 https://www.NetPad.net.cn/resource_web/course/#/533774.

Obviously, the value of “A” refers to the sum of radii of the two circles or the farthest distance away from the pole, so “A” can be any real number. Extreme values A or −A is reached when $x = (kpi - \varphi)/\omega$, where (k ∈ Z); and one extreme value corresponds to a petal. The whole graph axially stretches from the pole as parameter A increasing, and parameter “A” only affects the size of the graph.

With different “φ”, points start to move at different positions, causing the whole graph rotates about the pole at a certain angle. Meanwhile, by recalling corresponding knowledge about cosine curve in the rectangular coordinate system, students can draw the conclusion that the parameter “φ” decides the phase angle φ/ω of the initial phase, and “φ” can be a real number. Hence, the rose curve rotates at an angle of $-\varphi/\omega$ according to “φ”, and “φ” does not affect the shape of the whole graph.

By observing the construction with different “ω”, students would find out that points run at different speeds, and various numbers of petals appear. By referring to the period of the cosine curve in the rectangular coordinate system (the length of the period is 2pi/ω, the interval of the period is [−φ/ω + 2kpi/ω, (2pi − φ)/ω + 2kpi/ω)], (k ∈ Z), and ω ∈ R), students can set θ ∈ [−φ/ω, (2pi − φ)/ω), θ ∈ [−φ/ω, (2pi − φ)/ω + 2pi/ω)… to see what the graph of $\rho = A \cdot cos(\omega\theta + \varphi)$ looks like.

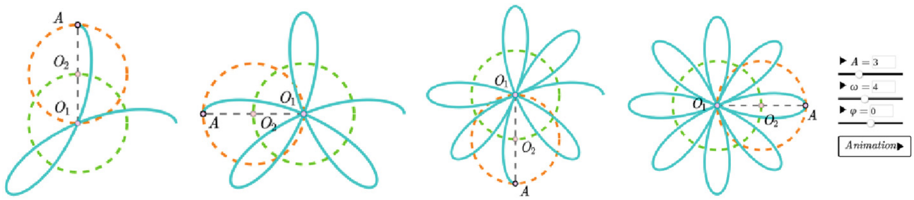

Fig. 6. Petals corresponding to different periods

As shown in Fig. 6, there are “two complete petals” in one period, “four” in two periods, “six” in three periods…. Since the rose curve is a closed curve, if there are just P complete periods (2pi/ω) within an interval of Q * 2pi, then the rose curve is closed (where P and Q are irreducible integers). That is, $P * 2pi/\omega = Q * 2pi$, i.e., $\omega = P/Q$; and “ω” should be set as a rational number for the graph of polar equation $\rho = A \cdot cos(\omega\theta + \varphi)$, which also can be represented as $\rho = A \cdot cos(\frac{P}{Q}\theta + \varphi)$.

According to the aforementioned analysis, students may choose their own model, we get the model containing “two secant circular motions” and corresponding parameter ranges, as the courseware[5] shown in Fig. 7. On the basis of the model, students can further dig the most characteristic feature of the rose curve, i.e., the number of petals, so as to draw various flowers mathematically as they like.

[5] https://www.NetPad.net.cn/resource_web/course/#/536844.

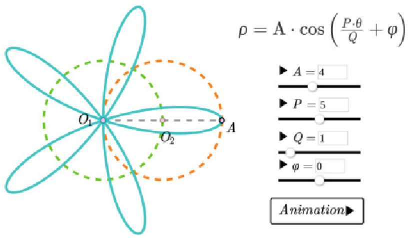

Fig. 7. The model of "secant circular motions"

4 Explore the Characteristic Feature - Petals

4.1 Draw More Rose Curves, Observe and Summary

On the basis of the aforementioned construction, students can draw more graphs by dragging the sliders and observe the relationship between the coefficient and the number of petals, as shown in Fig. 8.

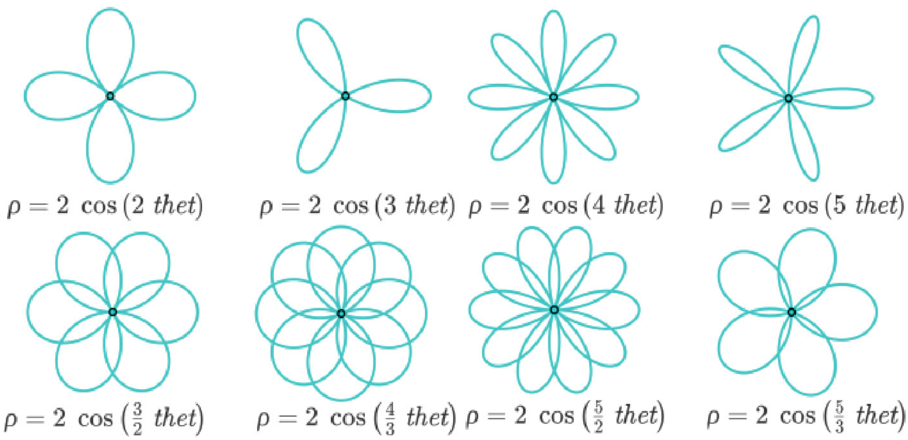

Fig. 8. Rose curves with various petals

Following facts would be easily got:

- The petals do not partly overlap with each other when P/Q is an integer; and the petals partly overlapped with each other when P/Q is a fraction.
- There are P petals when P/Q is odd, and there are 2P petals when P/Q is even.
- For a fraction P/Q, there are 2P petals when only one of P and Q is even, and there are P petals when both P and Q are odd.

To find out why, we can list out the polar coordinates for each peak by taking the equation $\rho = A \cdot cos(\frac{P}{Q}\theta + \varphi)$ as an example. All peaks are numbered from 0 to 2P-1;

peaks can be reached when θ = −φ/ω, (pi − φ)/ω, (2pi − φ)/ω, (3pi − φ)/ω, (4pi − φ)/ω…; and polar coordinates of each peak can be listed as (A, − φ/ω), (−A, (2pi − φ)/ω), (A, (3pi − φ)/ω), (−A, (4pi − φ)/ω)…. Moreover, in the polar coordinate system, a point with a negative radius is exactly the same point as one with the same positive radius, but in the opposite direction from the pole, that is, (−a, θ) is the same point as (a, θ + pi). Thus, our list of the peaks of the rose curve can be organized into Table 1.

Obviously, for even P, Q only can be odd, and the last P peaks do not overlap the first P petals; for odd P, Q could be even or odd, when Q is an even number, the last P peaks do not overlap the first P petals, and when Q is an odd number, last P peaks overlap the first P petals. Meanwhile, students can find the rule of the period of the whole graph, that is, when only one of P and Q is even, the period is 2Q * pi, or the period is Q * pi.

Table 1. List of Peaks

N^{th} peak	θ	Peak Coordinate		Organized Peak Coordinate	
0	$-\frac{\varphi}{\omega}$	$(A, -\frac{\varphi}{\omega})$		$(A, -\frac{\varphi}{\omega})$	
1	$\frac{\pi-\varphi}{\omega}$	$(A, \frac{\pi-\varphi}{\omega}+\pi)$		$(A, \frac{\pi}{\omega}-\frac{\varphi}{\omega}+\pi)$	
2	$\frac{2\pi-\varphi}{\omega}$	$(A, \frac{2\pi-\varphi}{\omega})$		$(A, \frac{2\pi}{\omega}-\frac{\varphi}{\omega})$	
...	...	...		...	
		Even P	**Odd P**	**Even P**	**Odd P**
P − 1	$\frac{(P-1)\pi-\varphi}{\omega}$	$(A, \frac{(P-1)\pi-\varphi}{\omega}+\pi)$	$(A, \frac{(P-1)\pi-\varphi}{\omega})$	$(A, Q\pi-\frac{\pi}{\omega}-\frac{\varphi}{\omega}+\pi)$	$(A, Q\pi-\frac{\pi}{\omega}-\frac{\varphi}{\omega})$
P	$\frac{P\pi-\varphi}{\omega}$	$(A, \frac{P\pi-\varphi}{\omega})$	$(A, \frac{P\pi-\varphi}{\omega}+\pi)$	$(A, Q\pi-\frac{\varphi}{\omega})$	$(A, Q\pi-\frac{\varphi}{\omega}+\pi)$
P+1	$\frac{(P+1)\pi-\varphi}{\omega}$	$(A, \frac{(P+1)\pi-\varphi}{\omega}+\pi)$	$(A, \frac{(P+1)\pi-\varphi}{\omega})$	$(A, Q\pi+\frac{\pi}{\omega}-\frac{\varphi}{\omega}+\pi)$	$(A, Q\pi+\frac{\pi}{\omega}-\frac{\varphi}{\omega})$
P+2	$\frac{(P+2)\pi-\varphi}{\omega}$	$(A, \frac{(P+2)\pi-\varphi}{\omega})$	$(A, \frac{(P+2)\pi-\varphi}{\omega}+\pi)$	$(A, Q\pi+\frac{2\pi}{\omega}-\frac{\varphi}{\omega}+\pi)$	$(A, Q\pi+\frac{2\pi}{\omega}-\frac{\varphi}{\omega})$
...	...	...	...	...	...
2P-1	$\frac{(2P-1)\pi-\varphi}{\omega}$	$(A, \frac{(2P-1)\pi-\varphi}{\omega}+\pi)$		$(A, 2Q\pi-\frac{\pi}{\omega}-\frac{\varphi}{\omega}+\pi)$	

4.2 Raise a New Question

After analyzing the effects of parameters P and Q, we can draw flowers having an expected number of petals according to the rule. However, it seems that we can draw any flower but a flower that, petals of which do not partly overlap with each other, and the number of the petals is an even number but not a multiple of four, such as six petals, ten petals and so on (Fig. 9). Shall we make a further step to make it out? Of course, but we need to go back to the very beginning, that is, the equation.

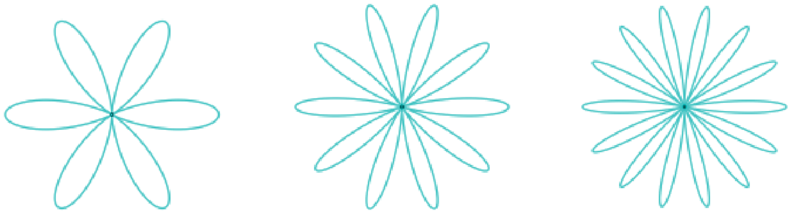

Fig. 9. Flowers cannot be generated according to equation $\rho = A \cdot cos(\omega\theta + \varphi)$

5 Draw the Generalized Rose Curve

Since a more generalized form of $y = A \cdot cos(\omega x + \varphi)$ is $y = A \cdot cos(\omega x + \varphi) + B$ in the rectangular coordinate system, we are inspired to explore the more generalized form of cosine curve $\rho = A \cdot cos(\omega\theta + \varphi) + B$ in the polar system.

5.1 Analyze and Graph Equation $\rho = A \cdot cos(\omega\theta + \varphi) + B$

The first step is also to analyze the equation and find out whether the graph of the polar equation $\rho = A \cdot cos(\omega\theta + \varphi) + B$ is generated by some points moved in certain forms. By transforming it to rectangular coordinate form, converting, and organizing, the polar equation $\rho = A \cdot cos(\omega\theta + \varphi) + B$ is converted into the following form:

$$\begin{cases} x = \frac{A}{2}[cos((\frac{P}{Q} + 1)\theta + \varphi) + cos((1 - \frac{P}{Q})\theta - \varphi)] + B \cdot cos\theta \\ y = \frac{A}{2}[sin((\frac{P}{Q} + 1)\theta + \varphi) + sin((1 - \frac{P}{Q})\theta - \varphi)] + B \cdot sin\theta \end{cases} \quad (5)$$

According to the equations, the graph is generated by a point driven by a sum of three different circular motions, and the circular motions have different radius, moving speeds, and phrase angles. Based on the previous exploration experience, or by referring to the scenarios of three circular motions that occur in life, such as the moon travels around the earth while they together travel around the sun, students can construct the model[6] shown in Fig. 10: with the animation function provided by Nepad, point O_2 moves according to the equation in light green, point O_3 moves according to the equation in dark green, and point A moves according to the equation in orange, where A ∈ R, B ∈ R, P ∈ N, Q ∈ N, and φ ∈ [0, 2pi).

5.2 Observe and Summarize Effects of Parameters

On the basis of the aforementioned construction, it is obvious that the length of the petal is A + B or A − B, which exactly corresponds to extreme values of equation $\rho = A \cdot cos(\omega\theta + \varphi) + B$. The interesting thing is that, if |A| > |B|, there are small petals generated in different directions, then students would be motivated to observe and make a classified discussion about how A and B affect the petals. Since the parameter B only determines the size of the curve so it can be a real number.

6 https://www.NetPad.net.cn/resource_web/course/#/496307.

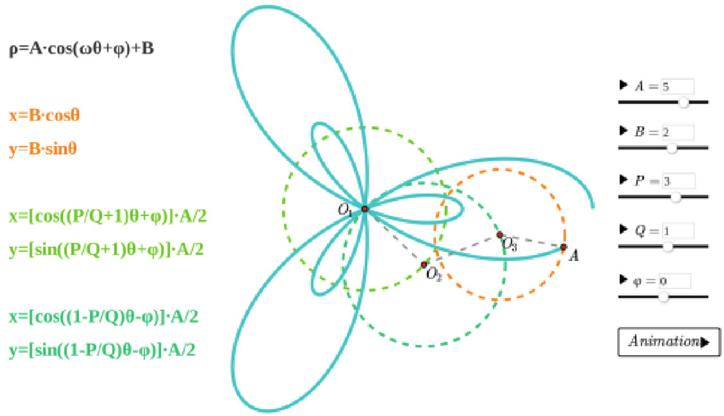

Fig. 10. A model according to equation $\rho = A \cdot \cos(\omega\theta + \varphi) + B$ (Color figure online)

- $|A| < |B|$: petals are not gathered together at one point but connected inwards a circle of diameter $|A - B|$ and outwards a circle of diameter $|A + B|$, shown in Fig. 11.

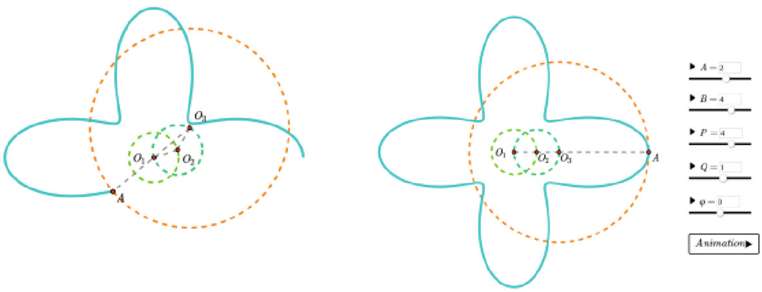

Fig. 11. A case when $|A| < |B|$

- $|A| = |B|$: petals are gathered together at a point and connected outwards a circle of diameter $A + B$, as shown in Fig. 12.

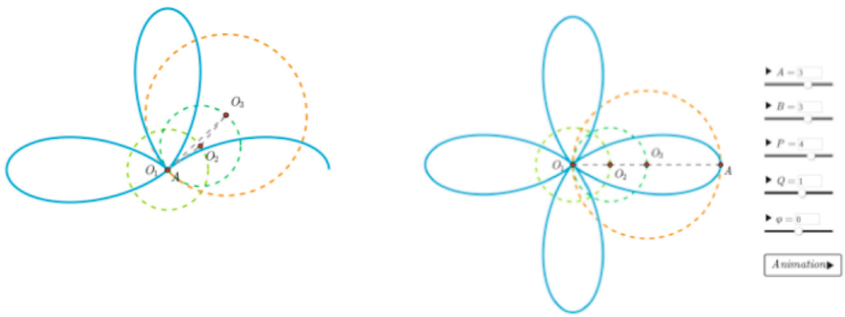

Fig. 12. A case when |A| = |B|

- |A| > |B|: there are smaller petals and bigger petals, all petals are gathered together at a point, but smaller petals are connected outwards a circle of diameter |A − B| and bigger petals are connected outwards a circle of diameter |A + B|, as shown in Fig. 13. In addition, all petals have the same size when B = 0, this is exactly the situation of $\rho = A \cdot cos(\omega\theta + \varphi)$.

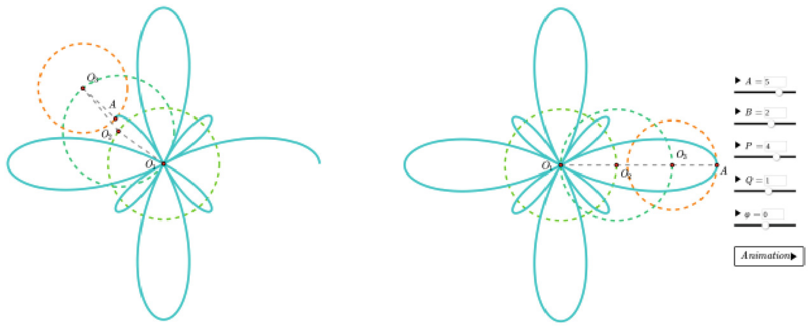

Fig. 13. A case when |A| > |B|

On the basis above, students can observe and summarize by graphing various rose curves, and finally draw a conclusion shown in Table 2. With this exploring process, they can mathematically draw flowers having desired number and shape of petals.

Table 2. Rules of Number and Shapes of Petals

<table>
<tr><th colspan="2">P/Q</th><th>|A|=<|B|</th><th>|A|>|B|</th><th>|B|=0</th></tr>
<tr><td>P/Q=integer</td><td>P/Q=odd</td><td>P petals</td><td>2P petals</td><td>P petals</td></tr>
<tr><td>Petals without overlap</td><td>P/Q=even</td><td>P petals</td><td>2P petals</td><td>2P petals</td></tr>
<tr><td>P/Q=fraction</td><td>P=odd & Q=odd</td><td>P petals</td><td>2P petals</td><td>P petals</td></tr>
<tr><td>Petals with overlap</td><td>P=even or Q=even</td><td>P petals</td><td>2P petals</td><td>2P petals</td></tr>
</table>

*For different values of P, Q, A, and B, the number of petals (in blue) are different

6 Conclusion

This learning activity explores the correspondence between the number of petals and parameters of the equation, by personally experiencing the whole process of discovering mathematical knowledge in the classroom with the support of information technology, students not only master the specific mathematical knowledge but also develop their mathematical learning and exploring skills.

Specifically, on the basis of students' prior knowledge about the method of constructively graphing cosine equations in rectangular coordinates system, they are encouraged to constructively graph polar equation $\rho = Acos(\omega\theta + \varphi)$ with analogical, associative, and innovative abilities, thus geometric meanings of parameters in the polar coordinate equation can be intuitively and profound understood after the constructive operations; furthermore, by graphing a number of rose curves, students observe, summary, express, and verify the rule of the petals; meanwhile, a new problem is raised, i.e., there are still some graphs that cannot be drawn, by logically guessing, students are motivated to graph polar equation $\rho = Acos(\omega\theta + \varphi)$ with the similar constructive method in NetPad, and finally draw a more complete conclusion about the rule of the petals. At last, students achieve the goal of mathematically drawing various flowers having the desired number of petals, during which they can experience the thinking and process of mathematicians inventing or discovering new knowledge, and further develop their innovation awareness and practical ability.

At the same time, information technology, as a beneficial tool for mathematical inquiry learning, provides a very ideal environment for students to actively discover and explore problems. As in the activity, students can creatively draw and set different circular motions to graph equation $\rho = Acos(\omega\theta + \varphi)$, conveniently observe various graphs by dragging variable sliders, and quickly reconstruct a model to graph equation $\rho = Acos(\omega\theta + \varphi) + B$. Furthermore, students can explore the two circular motions in the ladder model after this activity, and further draw various rose curves based on the ladder model. Hence, there is a larger space provided for students to practice or realize their ideas by doing mathematical experiments on NetPad, so that they can obtain real mathematical experiences rather than abstract mathematical conclusions. NetPad, an information technology that is deeply associated with maths subjects, balances abstract thinking and visual thinking, hands-on operation and mental work, as well as independent thinking and cooperative communication, making maths learning a more complete cognitive process. Moreover, the use of information technology does not cover up the thinking process, but constructively dissects the geometric principles, further making it a proper foundation for STEM education, etc.

In the future, it is expected to carry out a formal empirical study, so as to compare with traditional inquiry-based maths learning, quantitatively analyze the specific impact of information technology on inquiry-based mathematics learning, and evaluate students' exactly learning outcomes.

Acknowledgements. This Work is partially supported by the Innovation Research for the Full-time Postgraduates of Guangzhou University (No. 2022GDJC-M34), the China Postdoctoral Science Foundation (No. 2021M700920), the Guangzhou Academician and Expert Workstation (No. 20200115-9), and the Innovation Projects of Universities in Guangdong Province (No. 2020KTSCX215).

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